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Some Paraconsistent and Paracomplete Logics

The aim of my paper is to be outlined the specific features of some paraconsistent and paracomplete logics and their relationship to Beall and Restall's interpretation of logical pluralism.

Contemporary logic is characterized with a plurality of logical systems that puts the questions: which logical systems explicate optimally the process of logical thinking and express most accurately the essence of human rationality. There are different approaches trying to capture the universal features of logic.

For me is reasonable the following approach: to accent on this common and specific basis of all logical systems which is a warrant to identify them as logics¹ and which is in our intuitive idea for the nature of logical reasoning. I prefer this approach more than the comparison of theorems and the attempts to be reduced some systems to others (which is heuristic itself), because of the fact that some logical systems contain axioms which do not present in other ones; on the other hand most of the systems are set up on different principles, because of that their logical terms and theorems have totally different meaning. At the same time these logics are based on different philosophical and logical intuitions.

So, I stand on the position that logical consequence is this common core of logics, because on the one hand it is its main subject matter and on the other hand it is the stable base of any logical system. This standpoint is strongly embraced with Beall and Restall's view that logical consequence is the fundamental notion in logical systems. Actually it entitles them to be called logic (this circumstance defines the uniformity of logic, it is a reason to be said that logic is one) and at the same time is a reason for their difference because, in their words, the valid argument is one whose conclusion is true in every *case* in which all its premises are true (Beall&Restall, 2000, p.476); it explicates the pretheoretical notion of validity, which can be precise

¹ According to the methodological conception of logical fundamentalism, the different logics are only kind of logic and there must be something common between all of them (see Tabakov, 2000, p.16).

in different ways, because the different logics appear as different specifications of these *cases* (Beall&Restall, 2000, p.477).

Although when speaking about logical consequence I prefer to have in mind its two forms – the semantic and syntactic ones, I will further present Beall and Restall's notion of consequence, which they regard in model theoretic style, and will accent on its role for the plurality of logics moreover that it will also give me the opportunity to present some examples for inconsistent and incomplete "cases" (used by paraconsistent and respectively paracomplete systems), included in this definition of validity, and to say something about their epistemological significance. From another point of view the logical systems, as is known, are usually constructed in such a way that the syntactically valid formulas are at the same time semantically valid and vice versa, so to regard the semantic consequence, I think, is quite reasonable.

And so, Beall and Restall claim that there is only one notion of logical consequence and it is captured in the mentioned definition of the valid argument, but in the same time there are lots of true instances of this argument each of which specify different consequence relations - a reason for the plurality of logical systems. Hence, they are pluralists about logical consequence, which does not mean that they are pluralists about logic, understood as the study of consequence relations (Beall & Restall, 2006, p.88).

On the opposite extreme logical monists² assert that there is only one deductive consequence and the different logics do better or worse work, modelling this relation (Beall&Restall, 2005). According to logical contextualism or relativism the validity of argument depends on the context or on any other criterion. As pluralists about consequence, Beall and Restall are on the position that there are many consequence relations, each reflecting different precisifications of the pre-theoretic notion of deductive logical consequence (Beall & Restall, 2006, p.88). They give the following example: the inference from α to $\gamma \vee \sim\gamma$ is valid in the usual classical sense, because $\gamma \vee \sim\gamma$ is always true in classical logic, nevertheless what is the value of α . But the same argument is invalid in relevant logic, because there is not a content relation between the premise α and the conclusion $\gamma \vee \sim\gamma$ and we are not relevantly entitled to infer $\gamma \vee \sim\gamma$ from α (Beall&Restall, 2006, p.94).

² Graham Priest is monist in regard to logical consequence

A main accent here is that there is more than one notion of *case*, which can be substituted correctly in the definition of validity and there are at least three adequate logics presenting sufficient *cases* for the clarification of validity. The last are classical logic, relevant logic and intuitionistic logic, which are well motivated and express an own sense of *cases* appropriate for a substitution in the scheme of validity. The different logics, presenting different *cases*, are not rivals, because they give equally good specifications of the consequence relation (Beall & Restall, 2001, p.1-2). The reason is, as Restall and Beall show, that it can be supported the considerations, expressed by classical means, but also can be supported the considerations, presented by relevant logic or by intuitionistic logic. Although classical logic do not preserve the content relation between the premises and the conclusion due to its truth-functional character and the presence of the principle of compositionality, it preserves the truth, namely the classical valid argument cannot lead us from truth to falsity. The same holds for relevant logic as well as for intuitionistic logic whose valid arguments also do not lead us from truth to falsity, but at the same time which present different logics equally applicable for the analysis of reasoning as well as the classical one (Beall & Restall, 2001, p.2). Each of these logics clarifies the intuitive (pretheoretical) notion of logical consequence. For example, classical logic uses worlds (specifying the *cases*) where the laws of excluded middle and the law of non-contradiction are valid. In the semantics of relevant logic the worlds are often interpreted as situations (in Barwise –Perry style) where these two laws are not valid everywhere, because of that relevant logic is simultaneously inconsistent and incomplete. The *cases* in intuitionistic logic are constructions – the law of excluded middle is not valid here and therefore it is an incomplete logic³.

I am inclined to agree with Restall and Beall that these three logics present good *cases* for the clarification of the definition of valid logical argument and I will pay attention to inconsistent and incomplete worlds, used by some paraconsistent and paracomplete logics, as adequate and inadequate examples of these *cases*.

To me, from the family of paraconsistent and paracomplete logics, relevant logic is one of the logics offering most sufficiently the notion of consequence, both in semantic and syntactic plan. Relevant consequence is modeling the intuitive notion of consequence where the main accent is on the connection between the premises and

³ For more details see (Restall, 2001, p.2), (Beall & Restall, 2001, p.2), (Beall & Restall, 2000, p.13,15,17).

the conclusion, namely it preserves not only truth, but also - information. The principle of monotonicity is not entirely valid in relevant logic; the implication is not truth-functional in order to be closer to the conditional from natural reasoning (in R) and entailment (in E). These constraints are used to prevent a derivation of arbitrary conclusion from contradictory premises (because of that relevant logic is paraconsistent) as well as a derivation of tautology from arbitrary premises (which makes this logic paracomplete) contrary to classical logic where it actually happens. The norms used for achieving these aims reflect naturally on the semantics of relevant logic where to justify these requirements is necessary to be accepted the inconsistent and incomplete situations, namely such ones where the laws of non-contradiction and of the excluded middle are not valid everywhere. Although in the beginning the acceptance of such worlds (or situations) is due to some technical reasons, the investigations subsequently show that they can be interpreted in different ways depending on certain philosophical considerations. The last are usually epistemological, but there are also ontological and instrumentalist ones. I myself mostly respect the epistemological grounds because of which it can be found a lot of applications concerning logically undetermined concepts and inconsistent information. The inconsistent and incomplete worlds refer to many situations interpreted as limited parts of reality and can be sufficient means when there is a lack of information on some issue (represented by the incomplete worlds) or when this information is contradictory (represented by the inconsistent worlds). They give an opportunity to include the inconsistencies in our logical transformations and to work with them until precise the available information and differentiate exactly our contentions. On the other hand they give resources to be excluded those contentions which are indifferent for our system. Hence these cases are not only useful for the precisification of logical consequence, but also for lots of epistemological aims.

There are paraconsistent and paracomplete logics, however, which according to me, cannot pretend that are so much well motivated as relevant ones. The last means that actually as Beall and Restall's, I also have my own preferences in respect to the privilege of some systems in comparison with others, which don't contradict to the idea of logical pluralism.

I mean that not each paraconsistent or paracomplete logic present any interesting and well motivated aspects of *cases* though they surmount any shortcomings of classical consequence, for instance. To me supervaluationism and

subvaluationism are examples of such systems which are inadequate in many aspects. I mean the discrepancy between the preliminary considerations on the problem of vagueness which these logics attempt to solve and the desideratum to retain most aspects of classical semantics which in their case do not make a good job (I think); on the other hand I have lots of disagreements about the concrete logical apparatus which they use. From first sight it seems that subvaluationism and supervaluationism would propose good solutions for vague *cases*, namely for the usage of vague propositions and predicates, since they deny the principle of bivalence and assess the vague propositions as having the both truth values (the vague proposition is true and false simultaneously) for subvaluationism and respectively according to supervaluationism there is a lack of truth values for such propositions. But actually although the failure of bivalence these two logics retain the law of non-contradiction and respectively the law of excluded middle in their syntaxes. In this way the conjunction and disjunction are not truth-functional or more precisely they are sometimes truth-functional and sometimes they are not. According to supervaluationism, ‘ p or q ’ is sometimes true when no answer to the question ‘Which?’ is true. For similar reasons, ‘Something is F ’ is sometimes true when no answer to the question ‘Which thing is F ?’ is true. (Williamson, 1994, p.153). It has used extra-logical prerequisites without giving a clear mechanism how to assess the formulas in the different cases, namely it has proposed informal arguments to be solved a formal issue without clear principles and rules. In comparison with relevant logic, the last changes the meaning of implication (and respectively entailment) in order to accommodate the pre-theoretical intuition about consequence, but otherwise it retains the truth-functionality of disjunction and conjunction and (for me) it seems more adequate than the former two logics, although they are used for different aims.

Indeed supervaluationism and subvaluationism present different aspects of consequence as far as the principle of bivalence is denied in the both systems and in subvaluationism the spread consequence relation $A, \sim A \models B$ (known as the principle of explosion) fails, which makes the system paraconsistent; at the same time supervaluationism denies the contra intuitive relation $B \models A, \sim A$ (known as the principle of implosion), because of which the system is paracomplete⁴. But although these circumstances, they are only weakly paraconsistent and paracomplete logics.

⁴ For more information concerning these issues, see (Hyde, 2008, p.73-95).

The reason is, as I already mentioned, the retention of the law of non-contradiction and respectively the law of excluded middle. It means that although in borderline cases A and $\sim A$ can be simultaneously true (in subvaluationism), their conjunction $A \wedge \sim A$ is false; and nevertheless in borderline cases A and $\sim A$ have not truth values (in supervaluationism) $A \vee \sim A$ is still valid. The incoherence in the usage of some principles is counter-intuitive and undermines the underpinnings of these systems.

The mentioned incoherence appears also in regard to sorites paradoxes which subvaluationism and supervaluationism claim to solve. In the first one there is incoherence in the usage of modus ponens (the disjunctive syllogism) – sometimes it is valid and sometimes it is invalid. In the second one there is a discrepancy between the preliminary assumption that an unique sharp line between the predicate's positive and negative extension (of a vague term) do not exist and the usage of exactly the opposite circumstance (that somewhere there is such unique sharp line) in the treatment of this paradox.

All of this is a reason for me to claim that not all paraconsistent and paracomplete logics are sufficient for the purposes they pretend to realize. In regard to consequence, relevant logic is better than subvaluationism and supervaluationism, because it not only avoid the triviality of semantic consequence, but also do that on syntactic level, ensuring good inference standards in order to save the content relation between the premises and the conclusion. The last doesn't hold in subvaluationism and supervaluationism, because they use material implication and as a whole do not endeavour to improve the inference criterions. In regard to *cases*, I think that the analyses of vagueness made by these two logics is not sufficient to accommodate the vague terms in a logical theory which circumstance is combined with the treatment of vagueness in a way which makes the systems only weakly paraconsistent and respectively paracomplete. I think, as Hyde, that the strong paraconsistent or paracomplete response may contribute more to the analyses of vagueness and will be closer to our intuitions about vague predicates and propositions.

In the same line of thought I have similar arguments against another logic concerning vague *cases* using somehow inconsistent and incomplete worlds and where is presented different consequence relation. I mean fuzzy relevant logic, which in one of its two variants, is obtained since is fuzzified relevant world semantics (Priest, 2002, p.263), namely Priest transforms the relevant semantics into a fuzzy

logic, replacing the set of truth values $\{0, 1\}$ with the closed interval $[0, 1]$ (Priest, 2002, p.266). The interpretation is augmented with the operator $*$ (the Routley star) on worlds, as well as with the ternary relation R on worlds (Priest, 2002, p.267-269). But there are not any suggestions concerning the meaning of both of them. I think that fuzzy relevant logic presented in this variant doesn't help a lot for the clarification of the problem of vagueness (although it is inspired of it) because it doesn't give to the standard fuzzy logic new interpretation and new semantic value of vague propositions (for the last aim is used the standard for fuzzy logic continuum of truth values) and doesn't exemplify any new characteristics of vagueness. Maybe the reason is due to the circumstance that actually the specific for relevant logic α and α^* worlds which is expectable to be used in order to referee to any vague propositions and to be interpreted eventually as any worlds of vagueness are not used in this way – they are included here together with the ternary relation as technical means rather ensuring the non-triviality of inference. It seems to me that the proposed variant makes fuzzy logic more relevant than giving solutions to the problem of vagueness. The last, of course, is a sufficient condition to be scrutinized this logic and to be made more investigations concerning its principles. On the other hand it is not shown how to treat the sorites paradox by this apparatus. Well, since modus ponens for material implication (actually the disjunctive syllogism) is not valid here as it is not valid in relevant logic, maybe it could be used in harmony with the arguments from subvaluationism in regard to this issue. Thus, although fuzzy relevant logic includes α and α^* , which in relevant logic are used to accommodate inconsistent and incomplete worlds, the last don't find sufficient applications concerning vague *cases* which doesn't mean that any future inquiries will not contribute to this area and they (with any additional constraints or any changes in the interpretation) couldn't be helpful in the discussed problem field, not only for the non-triviality of inference as is here. So, this logic proposes interesting consequence relation, but insufficient interpretation of the *cases* included in this scheme of validity.

Therefore I agree with Beall and Restall that all consequence relations are equally logical (namely to be necessary, formal and normative), but not all of them are equally good for everything. On the other hand they remain the field of logical systems open for new and more adequate logics – it means that the process of constructing logical systems is not concluded once and forever and as they note the

availability of rational arguments which do not correspond to existing logic is a good reason to be constructed a new one (Beall & Restall, 2005). In this line of thought I agree that classical logic, intuitionistic logic and especially relevant logic present different, but important features of logical consequence. The last are due to the different requirements of deductive validity which one logic cannot suffice – some logics can do better work in regard to the different aspects of the *cases* included in the definition of consequence.

As a matter of fact Beall and Restall confess the opportunity to arise logical pluralism on different levels – depending on the domain of quantification, the rules of inference, the specific *cases* and so on (Beall & Restall, 2000, p.16)⁵. But is any formal system containing consequence relation logic? Logical consequence keeps the fundamental features of all logics; it is a necessary, but not sufficient condition to call any system logic. The sufficient condition can be expressed with some additional requirements. As they note, it has to be explained what are the exact cases (worlds, constructions, situations and etc.) and to be presented their metaphysical “history”, namely what kind of entities they are. The other requirement is to be given a definition for truth in a *case*. And at last it is necessary to be presented the cases as set-theoretic models of some kind (Beall & Restall, 2000, p.3). Just now a certain system can function as logic.

But nevertheless what the specific *cases* are: ersatz constructions (Lewis)⁶; elements of any set-theoretical model for which is stipulated to be under certain constraints (Mares, Restall, Varzi)⁷; abstract objects - which existence is inferred or abstracted in rational way (Zalta, Vander Laan, Barwise)⁸ and etc., I also as Restall and Beall deem that the *cases* in the definition of consequence express the main characteristics setting the basic and some of the most interesting differences between the systems. The reason, to me, is that they are connected with the precisification of consequence – the necessary requirement to form logic and are basic for the models – the sufficient condition to work this logic. They are that semantic foundation which presents some of the most essential and fruitful variety of logics.

⁵ It is a personal decision to choose the most appropriate criterions for the rise of pluralism.

⁶ See (Mares, 1997, p.517, p.520)

⁷ See (Mares, 1997), (Restall, 1997), (Varzi, 1997).

⁸ See (Barwise, 1997, p.491), (Zalta, 1997), (Vander Laan, 1997).

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